

## Worksheet 3B - Graph Sketching

Date \_\_\_\_\_ Period \_\_\_\_\_

**For each problem, find the open intervals where the function is increasing and decreasing.**

1)  $h(x) = 2x^2 + 8x + 8$

2)  $f = -2x^2 + 8x - 6$

3)  $g(w) = -w^2 - 6w - 5$

4)  $g(w) = 2w^2 + 16w + 34$

5)  $h(r) = -r^3 + 3r^2$

6)  $f = \frac{1}{r+1}$

$$7) f(r) = (3r + 6)^{\frac{1}{3}}$$

$$8) g(t) = -\frac{3}{t-1}$$

$$9) g = -\frac{6x^2 - 6}{x^3}$$

$$10) h(s) = \frac{1}{6}(s+1)^{\frac{7}{3}} - \frac{14}{3}(s+1)^{\frac{1}{3}} - 2$$

$$11) f(w) = -\frac{2}{w^2 - 4}$$

$$12) r = -s^4 - s^3 + 4s^2 - 1$$

For each problem, find the open intervals where the function is concave up and concave down.

13)  $f = -x^2 - 2x - 1$

14)  $h(t) = 2t^2 - 8t + 10$

15)  $t = -x^2 + 4x + 1$

16)  $h(r) = -\frac{r^2}{2} - 2r - 2$

17)  $f(t) = t^3 - 2t^2 + 4$

18)  $h = (5t + 25)^{\frac{1}{2}}$

$$19) f(s) = -(2s - 4)^{\frac{1}{3}}$$

$$20) g(x) = -(-x + 5)^{\frac{1}{2}}$$

$$21) h(r) = -r^5 + 3r^3 + 1$$

$$22) h(x) = x^5 - 2x^3 + 4$$

$$23) y = -\frac{1}{5}(t + 3)^{\frac{5}{3}} + 2(t + 3)^{\frac{2}{3}}$$

$$24) g = s^5 - 3s^3 + 2$$

**For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.**

25)  $y = -x^2 - 2x - 2$

26)  $f(x) = 2x^2 - 8x + 3$

27)  $y = \frac{x^2}{2} - 3x - \frac{1}{2}$

28)  $y = \frac{x^2}{2} + x + \frac{1}{2}$

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

$$29) y = -(-4x + 8)^{\frac{1}{2}}$$

$$30) y = -\frac{x^3}{3} + \frac{2x^2}{3} - \frac{x}{3}$$

$$31) y = -\frac{x^2}{3x - 6}$$

$$32) y = -(x - 5)^{\frac{1}{3}}$$

$$33) y = \frac{x^2}{4x - 4}$$

$$34) y = -\frac{x^2}{2x - 2}$$

$$35) y = \frac{x^3}{6} - \frac{x^2}{6} - \frac{4x}{3}$$

$$36) f(x) = \frac{x^3}{6} + \frac{x^2}{2}$$

$$37) f(x) = -\frac{x^3}{3} - x^2$$

$$38) y = -\frac{1}{x+3}$$

$$39) y = -x^4 + 4x^2$$

$$40) f(x) = \frac{1}{6}(x-1)^{\frac{7}{3}} - \frac{14}{3}(x-1)^{\frac{1}{3}}$$

$$41) y = \frac{3}{16}(x+1)^{\frac{4}{3}} - \frac{3}{2}(x+1)^{\frac{1}{3}}$$

$$42) f(x) = -\frac{9x^2 - 9}{x^3}$$

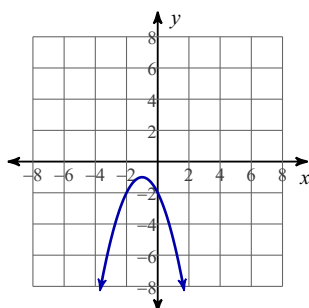
$$43) y = -\frac{x^4}{8} + \frac{x^2}{8}$$

$$44) f(x) = -\left(\frac{x+2}{x-3}\right)^2$$



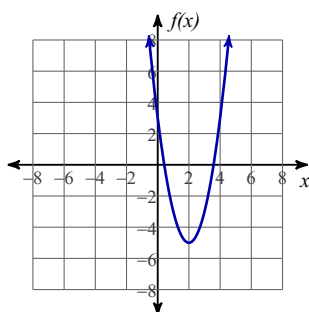
## Answers to Worksheet 3B - Graph Sketching (ID: 1)

- 1) Increasing:  $(-2, \infty)$  Decreasing:  $(-\infty, -2)$       2) Increasing:  $(-\infty, 2)$  Decreasing:  $(2, \infty)$   
 3) Increasing:  $(-\infty, -3)$  Decreasing:  $(-3, \infty)$       4) Increasing:  $(-4, \infty)$  Decreasing:  $(-\infty, -4)$   
 5) Increasing:  $(0, 2)$  Decreasing:  $(-\infty, 0), (2, \infty)$   
 6) Increasing: No intervals exist. Decreasing:  $(-\infty, -1), (-1, \infty)$   
 7) Increasing:  $(-\infty, \infty)$  Decreasing: No intervals exist.  
 8) Increasing:  $(-\infty, 1), (1, \infty)$  Decreasing: No intervals exist.  
 9) Increasing:  $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$  Decreasing:  $(-\sqrt{3}, 0), (0, \sqrt{3})$   
 10) Increasing:  $(-\infty, -3), (1, \infty)$  Decreasing:  $(-3, 1)$   
 11) Increasing:  $(0, 2), (2, \infty)$  Decreasing:  $(-\infty, -2), (-2, 0)$   
 12) Increasing:  $(-\infty, \frac{-3 - \sqrt{137}}{8}), (0, \frac{-3 + \sqrt{137}}{8})$  Decreasing:  $(\frac{-3 - \sqrt{137}}{8}, 0), (\frac{-3 + \sqrt{137}}{8}, \infty)$   
 13) Concave up: No intervals exist. Concave down:  $(-\infty, \infty)$   
 14) Concave up:  $(-\infty, \infty)$  Concave down: No intervals exist.  
 15) Concave up: No intervals exist. Concave down:  $(-\infty, \infty)$   
 16) Concave up: No intervals exist. Concave down:  $(-\infty, \infty)$   
 17) Concave up:  $(\frac{2}{3}, \infty)$  Concave down:  $(-\infty, \frac{2}{3})$   
 18) Concave up: No intervals exist. Concave down:  $(-5, \infty)$   
 19) Concave up:  $(2, \infty)$  Concave down:  $(-\infty, 2)$   
 20) Concave up:  $(-\infty, 5)$  Concave down: No intervals exist.  
 21) Concave up:  $(-\infty, -\frac{3\sqrt{10}}{10}), (0, \frac{3\sqrt{10}}{10})$  Concave down:  $(-\frac{3\sqrt{10}}{10}, 0), (\frac{3\sqrt{10}}{10}, \infty)$   
 22) Concave up:  $(-\frac{\sqrt{15}}{5}, 0), (\frac{\sqrt{15}}{5}, \infty)$  Concave down:  $(-\infty, -\frac{\sqrt{15}}{5}), (0, \frac{\sqrt{15}}{5})$   
 23) Concave up:  $(-\infty, -5)$  Concave down:  $(-5, -3), (-3, \infty)$   
 24) Concave up:  $(-\frac{3\sqrt{10}}{10}, 0), (\frac{3\sqrt{10}}{10}, \infty)$  Concave down:  $(-\infty, -\frac{3\sqrt{10}}{10}), (0, \frac{3\sqrt{10}}{10})$   
 25)



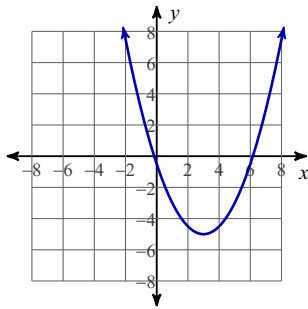
No x-intercepts. y-intercept at  $y = -2$   
 Critical point at:  $x = -1$   
 Increasing:  $(-\infty, -1)$  Decreasing:  $(-1, \infty)$   
 No inflection points exist.  
 Concave up: No intervals exist. Concave down:  $(-\infty, \infty)$   
 No relative minima. Relative maximum:  $(-1, -1)$

26)



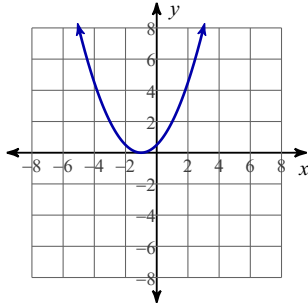
x-intercepts at  $x = \frac{4 - \sqrt{10}}{2}, \frac{4 + \sqrt{10}}{2}$  y-intercept at  $y = 3$   
 Critical point at:  $x = 2$   
 Increasing:  $(2, \infty)$  Decreasing:  $(-\infty, 2)$   
 No inflection points exist.  
 Concave up:  $(-\infty, \infty)$  Concave down: No intervals exist.  
 Relative minimum:  $(2, -5)$  No relative maxima.

27)



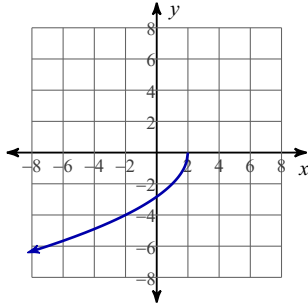
x-intercepts at  $x = 3 - \sqrt{10}, 3 + \sqrt{10}$  y-intercept at  $y = -\frac{1}{2}$   
 Critical point at:  $x = 3$   
 Increasing:  $(3, \infty)$  Decreasing:  $(-\infty, 3)$   
 No inflection points exist.  
 Concave up:  $(-\infty, \infty)$  Concave down: No intervals exist.  
 Relative minimum:  $(3, -5)$  No relative maxima.

28)



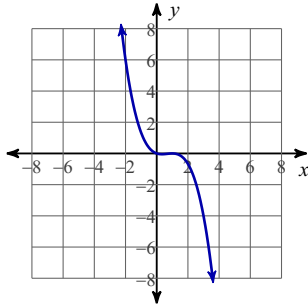
x-intercept at  $x = -1$  y-intercept at  $y = \frac{1}{2}$   
 Critical point at:  $x = -1$   
 Increasing:  $(-1, \infty)$  Decreasing:  $(-\infty, -1)$   
 No inflection points exist.  
 Concave up:  $(-\infty, \infty)$  Concave down: No intervals exist.  
 Relative minimum:  $(-1, 0)$  No relative maxima.

29)



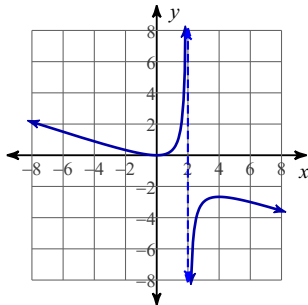
x-intercept at  $x = 2$  y-intercept at  $y = -2\sqrt{2}$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical point at:  $x = 2$   
 Increasing:  $(-\infty, 2)$  Decreasing: No intervals exist.  
 No inflection points exist.  
 Concave up:  $(-\infty, 2)$  Concave down: No intervals exist.  
 No relative minima. No relative maxima.

30)



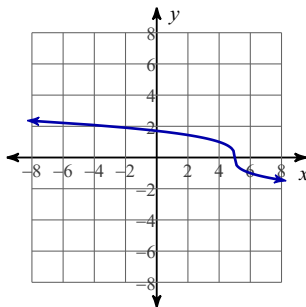
x-intercepts at  $x = 0, 1$  y-intercept at  $y = 0$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical points at:  $x = \frac{1}{3}, 1$   
 Increasing:  $(\frac{1}{3}, 1)$  Decreasing:  $(-\infty, \frac{1}{3}), (1, \infty)$   
 Inflection point at:  $x = \frac{2}{3}$   
 Concave up:  $(-\infty, \frac{2}{3})$  Concave down:  $(\frac{2}{3}, \infty)$   
 Relative minimum:  $(\frac{1}{3}, -\frac{4}{81})$  Relative maximum:  $(1, 0)$

31)



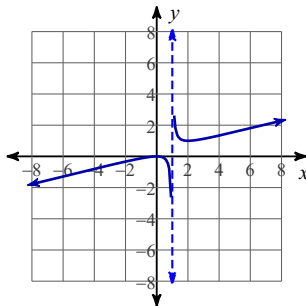
x-intercept at  $x = 0$  y-intercept at  $y = 0$   
 Vertical asymptote at:  $x = 2$   
 No horizontal asymptotes exist.  
 Slant asymptote:  $y = -\frac{x}{3} - \frac{2}{3}$   
 Critical points at:  $x = 0, 4$   
 Increasing:  $(0, 2), (2, 4)$  Decreasing:  $(-\infty, 0), (4, \infty)$   
 No inflection points exist.  
 Concave up:  $(-\infty, 2)$  Concave down:  $(2, \infty)$   
 Relative minimum:  $(0, 0)$  Relative maximum:  $(4, -\frac{8}{3})$

32)



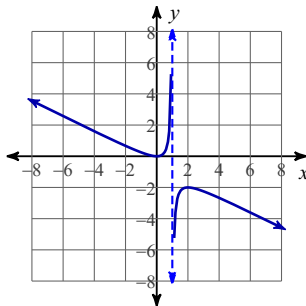
x-intercept at  $x = 5$  y-intercept at  $y = \sqrt[3]{5}$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical point at:  $x = 5$   
 Increasing: No intervals exist. Decreasing:  $(-\infty, \infty)$   
 Inflection point at:  $x = 5$   
 Concave up:  $(5, \infty)$  Concave down:  $(-\infty, 5)$   
 No relative minima. No relative maxima.

33)



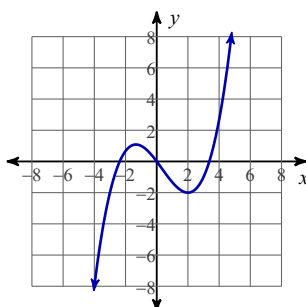
x-intercept at  $x = 0$  y-intercept at  $y = 0$   
 Vertical asymptote at:  $x = 1$   
 No horizontal asymptotes exist.  
 Slant asymptote:  $y = \frac{x}{4} + \frac{1}{4}$   
 Critical points at:  $x = 0, 2$   
 Increasing:  $(-\infty, 0), (2, \infty)$  Decreasing:  $(0, 1), (1, 2)$   
 No inflection points exist.  
 Concave up:  $(1, \infty)$  Concave down:  $(-\infty, 1)$   
 Relative minimum:  $(2, 1)$  Relative maximum:  $(0, 0)$

34)



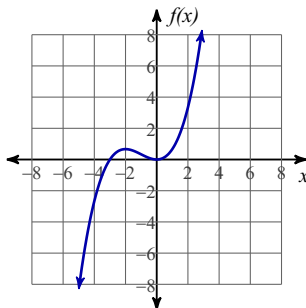
x-intercept at  $x = 0$  y-intercept at  $y = 0$   
 Vertical asymptote at:  $x = 1$   
 No horizontal asymptotes exist.  
 Slant asymptote:  $y = -\frac{x}{2} - \frac{1}{2}$   
 Critical points at:  $x = 0, 2$   
 Increasing:  $(0, 1), (1, 2)$  Decreasing:  $(-\infty, 0), (2, \infty)$   
 No inflection points exist.  
 Concave up:  $(-\infty, 1)$  Concave down:  $(1, \infty)$   
 Relative minimum:  $(0, 0)$  Relative maximum:  $(2, -2)$

35)



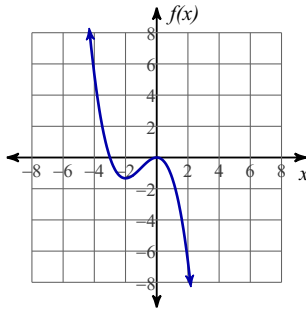
x-intercepts at  $x = \frac{1 - \sqrt{33}}{2}, 0, \frac{1 + \sqrt{33}}{2}$  y-intercept at  $y = 0$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical points at:  $x = -\frac{4}{3}, 2$   
 Increasing:  $(-\infty, -\frac{4}{3}), (2, \infty)$  Decreasing:  $(-\frac{4}{3}, 2)$   
 Inflection point at:  $x = \frac{1}{3}$   
 Concave up:  $(\frac{1}{3}, \infty)$  Concave down:  $(-\infty, \frac{1}{3})$   
 Relative minimum:  $(2, -2)$  Relative maximum:  $(-\frac{4}{3}, \frac{88}{81})$

36)



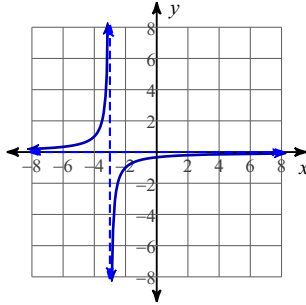
x-intercepts at  $x = -3, 0$  y-intercept at  $y = 0$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical points at:  $x = -2, 0$   
 Increasing:  $(-\infty, -2), (0, \infty)$  Decreasing:  $(-2, 0)$   
 Inflection point at:  $x = -1$   
 Concave up:  $(-1, \infty)$  Concave down:  $(-\infty, -1)$   
 Relative minimum:  $(0, 0)$  Relative maximum:  $(-2, \frac{2}{3})$

37)



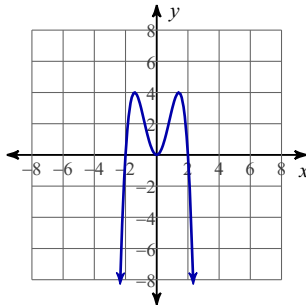
x-intercepts at  $x = -3, 0$  y-intercept at  $y = 0$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical points at:  $x = -2, 0$   
 Increasing:  $(-2, 0)$  Decreasing:  $(-\infty, -2), (0, \infty)$   
 Inflection point at:  $x = -1$   
 Concave up:  $(-\infty, -1)$  Concave down:  $(-1, \infty)$   
 Relative minimum:  $(-2, -\frac{4}{3})$  Relative maximum:  $(0, 0)$

38)



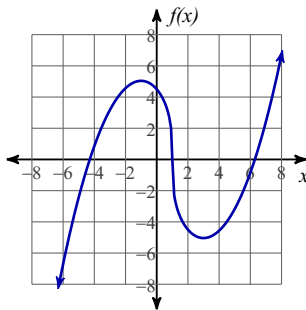
No x-intercepts. y-intercept at  $y = -\frac{1}{3}$   
 Vertical asymptote at:  $x = -3$   
 Horizontal asymptote at:  $y = 0$   
 No critical points exist.  
 Increasing:  $(-\infty, -3), (-3, \infty)$  Decreasing: No intervals exist.  
 No inflection points exist.  
 Concave up:  $(-\infty, -3)$  Concave down:  $(-3, \infty)$   
 No relative minima. No relative maxima.

39)



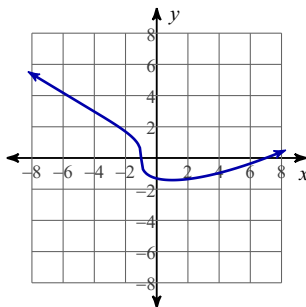
x-intercepts at  $x = -2, 0, 2$  y-intercept at  $y = 0$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical points at:  $x = -\sqrt{2}, 0, \sqrt{2}$   
 Increasing:  $(-\infty, -\sqrt{2}), (0, \sqrt{2})$  Decreasing:  $(-\sqrt{2}, 0), (\sqrt{2}, \infty)$   
 Inflection points at:  $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$   
 Concave up:  $(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3})$  Concave down:  $(-\infty, -\frac{\sqrt{6}}{3}), (\frac{\sqrt{6}}{3}, \infty)$   
 Relative minimum:  $(0, 0)$  Relative maxima:  $(-\sqrt{2}, 4), (\sqrt{2}, 4)$

40)



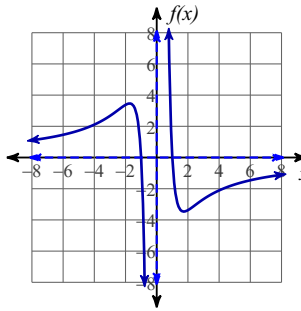
x-intercepts at  $x = 1 - 2\sqrt{7}, 1, 1 + 2\sqrt{7}$  y-intercept at  $y = \frac{9}{2}$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical points at:  $x = -1, 1, 3$   
 Increasing:  $(-\infty, -1), (3, \infty)$  Decreasing:  $(-1, 3)$   
 Inflection point at:  $x = 1$   
 Concave up:  $(1, \infty)$  Concave down:  $(-\infty, 1)$   
 Relative minimum:  $(3, -4\sqrt{2})$  Relative maximum:  $(-1, 4\sqrt{2})$

41)



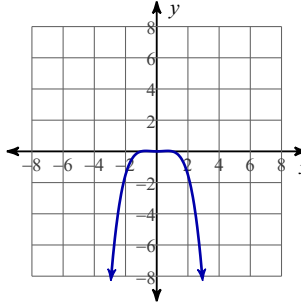
x-intercepts at  $x = -1, 7$  y-intercept at  $y = -\frac{21}{16}$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical points at:  $x = -1, 1$   
 Increasing:  $(1, \infty)$  Decreasing:  $(-\infty, 1)$   
 Inflection points at:  $x = -5, -1$   
 Concave up:  $(-\infty, -5), (-1, \infty)$  Concave down:  $(-5, -1)$   
 Relative minimum:  $(1, -\frac{9\sqrt{2}}{8})$  No relative maxima.

42)



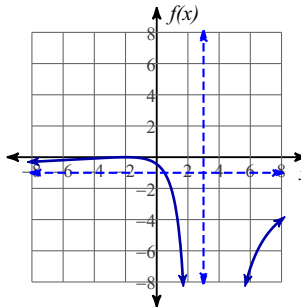
x-intercepts at  $x = -1, 1$  No y-intercepts.  
 Vertical asymptote at:  $x = 0$   
 Horizontal asymptote at:  $f = 0$   
 Critical points at:  $x = -\sqrt{3}, \sqrt{3}$   
 Increasing:  $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$  Decreasing:  $(-\sqrt{3}, 0), (0, \sqrt{3})$   
 Inflection points at:  $x = -\sqrt{6}, \sqrt{6}$   
 Concave up:  $(-\infty, -\sqrt{6}), (0, \sqrt{6})$  Concave down:  $(-\sqrt{6}, 0), (\sqrt{6}, \infty)$   
 Relative minimum:  $(\sqrt{3}, -2\sqrt{3})$  Relative maximum:  $(-\sqrt{3}, 2\sqrt{3})$

43)



x-intercepts at  $x = -1, 0, 1$  y-intercept at  $y = 0$   
 No vertical asymptotes exist.  
 No horizontal asymptotes exist.  
 Critical points at:  $x = -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}$   
 Increasing:  $(-\infty, -\frac{\sqrt{2}}{2}), (0, \frac{\sqrt{2}}{2})$  Decreasing:  $(-\frac{\sqrt{2}}{2}, 0), (\frac{\sqrt{2}}{2}, \infty)$   
 Inflection points at:  $x = -\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}$   
 Concave up:  $(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6})$  Concave down:  $(-\infty, -\frac{\sqrt{6}}{6}), (\frac{\sqrt{6}}{6}, \infty)$   
 Relative minimum:  $(0, 0)$  Relative maxima:  $(-\frac{\sqrt{2}}{2}, \frac{1}{32}), (\frac{\sqrt{2}}{2}, \frac{1}{32})$

44)



x-intercept at  $x = -2$  y-intercept at  $y = -\frac{4}{9}$   
 Vertical asymptote at:  $x = 3$   
 Horizontal asymptote at:  $f = -1$   
 Critical point at:  $x = -2$   
 Increasing:  $(-\infty, -2), (3, \infty)$  Decreasing:  $(-2, 3)$   
 Inflection point at:  $x = -\frac{9}{2}$   
 Concave up:  $(-\infty, -\frac{9}{2})$  Concave down:  $(-\frac{9}{2}, 3), (3, \infty)$   
 No relative minima. Relative maximum:  $(-2, 0)$