Calculus Names: Period:

Graphing Functions With Calculus

Derivatives contain considerable information about functions. The goal of this activity is to examine the graphs of functions and their derivatives to find patterns that will help us understand how to use derivatives to understand the behavior of functions.

I've provided a customized version of a "graphulator" for this assignment. This version of the graphulator contains two graphs, one that shows the original function and its first derivative, and another graph that shows the function and its second derivative. Work through the activity exercises below.

Please download and open U3 GRAPHULATOR.XLSX from the Chapter 3 page on my website (attached at the bottom of the page).

Make Sure You Can Use the Graphulator

1. Plot $y=x^2$ over the interval $x = [-2,4]$.

   ☐ Confirm that the graph and the x-values start at $x=-2$ and end at $x=4$.

Approximate the First Derivative (f')

2. Think about what the derivative really represents and come up with an Excel equation to approximate the derivative at any point x. You want to construct the derivative so that when you change the source function, the derivative is automatically calculated without you having to change the equation of the derivative. Explain how Excel approximates the derivative of a function in the space below.

   ☐ Check the graph and confirm your approximated derivative is correct. How will you know if it's correct?

Approximate the Second Derivative (f'')

3. Think about what the derivative really represents and come up with an Excel equation to approximate the second derivative at any point x. Explain how Excel approximates the derivative of a function in the space below.

   ☐ Check the graph and confirm your approximated second derivative is correct. How will you know if it's correct?

Calculus Names: Period:

How to Use the Graphulator

In this activity, you'll enter different functions into the graphulator and examine the first and second derivatives of the function relative to the function itself. You'll answer a series of questions that should help you analyze the relationship between the function and its derivatives. We'll use a set of functions to see what the first derivative tells us, then we'll use the set of functions again to see what the second derivative tells us. *Note: Excel is graphing a "window" of the entire graph. Assume the domain of the graph is* $(-\infty, +\infty)$.

Using the First Derivative

4. We'll use the spreadsheet to analyze $f(x) = 2x^3 - 9x^2$. Use *start* = -1 and *end* = 5

   ☐ Identify the approximate x-intervals over which the function is increasing and decreasing and write them below in the form (x_1, x_2). For each of the x-ranges you found, is y' positive or negative?

x-intervals	**Is $f(x)$ increasing or decreasing?**	$\frac{df}{dx} > 0$ **or** $\frac{df}{dx} < 0$?

5. Just based on these results, what information does the first derivative give you about the graph of a function?

6. Locate the relative maxima and minima for y and mark them on the table above. What is the value of the derivative at the extrema?

7. Now looking at your chart and the behavior of the derivative around the maxima and minima, how can you use the first derivative to tell the difference between a local maximum and a local minimum?

8. Let's check your understanding. **Without using Excel**, determine where the function $f(x) = 2x^3 - 3x^2 - 12x$ is increasing, and decreasing, and the locations of maximum and minimum values. Show and explain your work below. Try to make a sketch of the graph based on this information and without using technology.

9. Now check your work using Excel on [-2, 3] and sketch the graph. If your work above isn't corroborated by Excel, explain what you may have misunderstood or did wrong. If you have any questions or something is unclear at this point write them below and circle the questions so I know to follow-up with you.

Calculus Names: Period:

Using the Second Derivative

Now let's find out what the second derivative can tell us about a function.

Again consider $f(x) = 2x^3 - 3x^2 - 12x$ on [-2, 3]. Enter it into the Excel sheet and examine the graph of the function and its second derivative.

10. From our understanding of the first derivative, what are the interesting sub-intervals for $y(x)$we should consider? Enter them in the table below. The second derivative will give us different information, but in a manner similar to the first derivative. Observe the "shape" of the function – does it "open up", or "open down" in the intervals? How does this relate to the sign of y''? Enter your observations in the table below.

x-intervals	**Observations of** $f(x) = 2x^3 - 3x^2 - 12x$	**Sign of** $f''(x)$

11. In your own words, what information does the second derivative tell you about a function? How could you use the second derivative to find the maxima and minima for functions?

12. In the space below, using words and pictures, summarize your understanding of the role that the first and second derivative play in graphing functions. If you still have questions, write them down and circle them so I can follow up with you.