Euler's Method and Excel

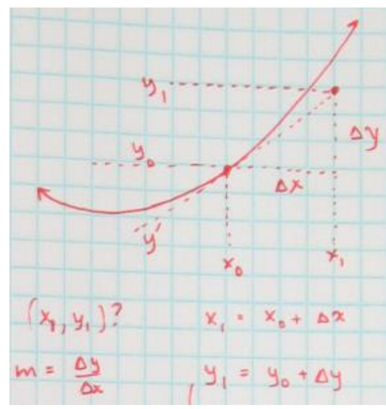
Introduction

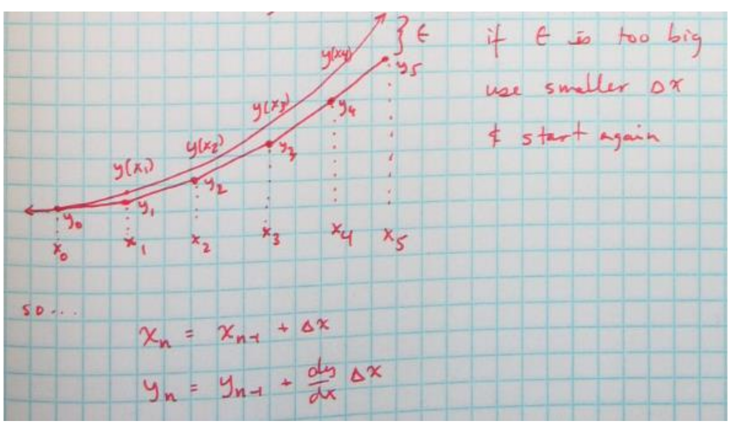
Euler's method allows us to find approximate solutions to differential equations. It was developed by Leonhard Euler in the middle of the 18th century. Although simple, good solutions required hundreds if not thousands of computations, which today are easily handled by computers. In this activity, you will use Excel to perform these computations to find graphical solutions to differential equations. After the activity, we'll explore a more complex spreadsheet that uses Euler's method to determine the trajectory of an object in flight in a vacuum and again taking air resistance into account.

Lots of Linear Approximations

If we "zoom-in" to a continuous region of a function, or in other words, examine the function over a very small interval, the function will look essentially like a straight line. If it doesn't, all we have to do is zoom in even further until it does. This property is called *local linearity*. Over that small interval, we can approximate the function as a straight line. If we move to the right or the left of that interval, the function will again look like a line, but it may have a different slope. This means that a true curving function can be approximated as a sequence of short, connected line segments, with the slop of each segment varying from segment to segment. Euler's method makes use of this approximation.

Consider the differential equation $\frac{dy}{dx} = y$. This DE tells us that the slope of the function at any point (*x,y*) is equal to the *y* coordinate. If the point is (1,3), then $\frac{dy}{dx} = 3$. So what is the value of the function at $x = 1.01$? The interval we're examining is $\Delta x = 0.01$. So if the slope of the the function is $m = \frac{dy}{dx} = 3$ then the approximate change in the vertical direction is $\Delta y = m\Delta x = \frac{dy}{dx}\Delta x = (3)(0.01) = 0.03$. So an approximate value for *y* when $x = 1.01$ is $y + \Delta y = 3.03$. Similarly, at $x = 0.99$, $\Delta x = -0.01$ so $\Delta y = \frac{dy}{dx}\Delta x = (3)(-0.01) = -0.03$, and the previous point is (0.99, 2.97). This is illustrated on the right.





Now that we know the next point is (1.01,3.03), then $\frac{dy}{dx} = y \approx 3.03$ so $\Delta y = 3.03\Delta x = .0303$, and we get (1.02, 3.0603). We can continue this process by using the current point to approximate the next point, then making that the current point, and so on. Since we're making only linear approximations, and then only keeping a certain number of digits, our accuracy will go down the further we project the approximation away from our starting point. However, if we want greater accuracy, we can just make our Δx smaller, and start over. This is illustrated on the left.

The complexity of this process is based on the complexity of $\frac{dy}{dx}$. Even if the derivative is complicated, all we have to do is plug-in the *x*- and *y*-values into the formula for the derivative get the slope of the linear approximation.

Euler's Method in Excel

Download `EULEREXPLORATION.XLSX` from my Post Test Activities page on my website. The first few lines of the spreadsheet are shown below.

Starting x	0	
Starting y	0	
Delta-x	0.1	
Solution for dy/dx =		
x	**dy/dx**	**y**
0		0
0.1		
0.2		
0.3		
0.4		

Do not change anying in the shaded cells. You can modify the unshaded cells.

The fields "Starting x" and "Starting y" represent your initial point on the solution curve (x_0, y_0).

Delta-x is your x-increment Δx.

Use the "Solution for dy/dx" field to enter your derivitive equation. To get started, just enter $2x$ as text. This field has an odd name because it's used as the title for the solution graph. Try to keep the title the same as the actual function you're graphing.

In the first unshaded cell under the"dy/dx" column enter the Excel equation that calculates $2x$.

In the first unshaded cell under the "y" column enter the Euler equation that calculates the approximate value of $y(0.1)$. Be careful. To calculate the current point, what values of *x* and *y* do we use?

Now fill the rest of the columns and look at your graph. Does the answer make sense for $\frac{dy}{dx} = 2x$?

If everything is working right, you should now only have to change the equation for *dy/dx*. Try $\frac{dy}{dx} = y$. What should the answer be? Does your graph match your answer?

Now play with different equations for *dy/dx* to make sure you're understanding how Euler's solution works. After some activity time, I will close the activity and will demonstarte a more complex problem that uses Euler's method.